LESSON 29

Probabilities of Simple Events

UNDERSTAND You do not need to perform an experiment to identify a probability. If you have knowledge about an event and its outcomes, you can create a **probability model**. This model assigns a theoretical probability to each outcome, according to this ratio:

$$\text{theoretical probability} = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

The favorable outcomes are the outcomes being considered. The set of all possible outcomes is called the **sample space**. The sum of the probabilities of all the outcomes is 1.

If each outcome in a chance process is equally likely, the probability model is called a **uniform probability model**.

Teresa flipped a fair coin 10 times. It landed heads up 6 times and tails up 4 times. What is the experimental probability of the coin landing heads up? What is the theoretical probability of the coin landing heads up? Are they the same or different? Explain.

1 Determine the experimental probability of the coin landing heads up.

$$\text{experimental probability} = \frac{\text{times event occurred}}{\text{total number of trials}} = \frac{6}{10} = \frac{3}{5}$$

2 Determine the theoretical probability of the coin landing heads up.
The sample space is: {heads, tails}. So, there are 2 possible outcomes.
There is 1 outcome being considered: heads. So, there is 1 favorable outcome.

$$\text{theoretical probability} = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}} = \frac{1}{2}$$

3 Are the two probabilities the same or different?
The two probabilities are different, because the experimental probability indicates how many times the coin actually landed heads up, 3 out of every 5 tosses, while the theoretical probability shows how many times it was expected to land heads up, 1 out of every 2 tosses.

▶ The theoretical and experimental probabilities may or may not be the same. In this case, the theoretical probability of the coin landing heads up is $\frac{1}{2}$, but the experimental probability of that outcome was $\frac{3}{5}$.

Connect

There are 24 students in a class. If a student is selected at random from the class, what is the probability that Anna will be selected? What is the probability that Hector will be selected? What kind of probability model is being used in this situation?

1 Identify the probability of any one student being selected.

One student will be selected, so there is one favorable outcome.

There are 24 students in the class, so there are 24 possible outcomes.

$$\text{theoretical probability} = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

$$\text{theoretical probability} = \frac{1}{24}$$

2 What is the probability, P, that Anna will be selected?

$P(\text{Anna}) = \frac{1}{24}$

P(Anna) is a way to represent the probability of selecting Anna.

3 What is the probability that Hector will be selected?

$P(\text{Hector}) = \frac{1}{24}$

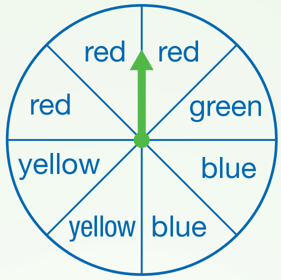
4 What kind of probability model is being used?

▶ Since selecting any student from the class is equally likely, the probability model for this process is uniform. The probability of selecting any single student, including Anna or Hector, is $\frac{1}{24}$.

TRY

If it is equally likely that a girl will be selected as it is that a boy will be selected in this situation, what is the theoretical probability of selecting a girl? What is the theoretical probability of selecting a boy? Explain.

EXAMPLE A This spinner is divided into 8 equal-sized sections. What is the probability of the spinner landing on each of the colors shown? Why is the probability model for this spinner **not** an example of a uniform probability model?



1 Identify the probability, P, of the spinner landing on each color.

There are 3 red sections. There are 8 sections in all.

$P(\text{red}) = \frac{3}{8}$

There is 1 green section, so:

$P(\text{green}) = \frac{1}{8}$

There are 2 blue sections, so:

$P(\text{blue}) = \frac{2}{8} = \frac{1}{4}$

There are 2 yellow sections, so:

$P(\text{yellow}) = \frac{2}{8} = \frac{1}{4}$

2 Explain why the probability model is not an example of a uniform probability model.

▶ A uniform probability assigns equal probability to each outcome. For this spinner, $P(\text{red}) = \frac{3}{8}$, $P(\text{green}) = \frac{1}{8}$, $P(\text{blue}) = \frac{1}{4}$, and $P(\text{yellow}) = \frac{1}{4}$, so the probabilities are not all equal.

TRY

If you were to spin the spinner shown above 120 times, how many times would you expect the spinner to land on red?

EXAMPLE B Two number cubes, each with faces numbered 1 to 6, are tossed at the same time. What is the probability of tossing a sum of 9?

1 Create a chart to show all the possible sums.

For example, the sum when a 1 is rolled on the first number cube and a 1 is rolled on the second number cube is: $1 + 1 = 2$.

First Number Cube

Second Number Cube	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	**9**
4	5	6	7	8	**9**	10
5	6	7	8	**9**	10	11
6	7	8	**9**	10	11	12

The diagram shows that there are 4 ways to toss a sum of 9. So, there are 4 favorable outcomes.

2 What is the probability of tossing a sum of 9?

There are 36 possible outcomes in the chart. Four of them are favorable.

$P(\text{a sum of } 9) = \frac{4}{36} = \frac{1}{9}$

TRY

What is the probability of tossing the two number cubes and rolling a sum greater than 9? Explain.

Practice

Each spinner shown is divided into equal-sized sections. Identify the theoretical probability of each spinner landing on a shaded section, *P*(shaded). Simplify, if possible.

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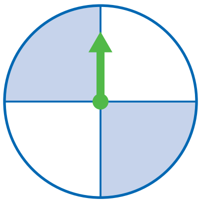

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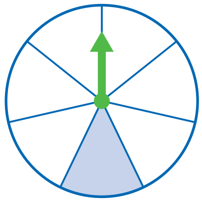

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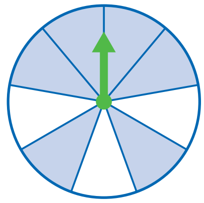

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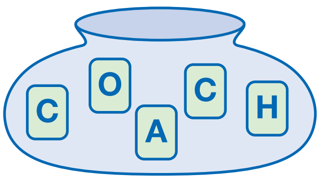
HINT: The shaded sections are the favorable outcomes.

Each bag contains lettered tiles, each the same size. Identify the theoretical probability of selecting the letter C from each bag. Simplify, if possible.

4.



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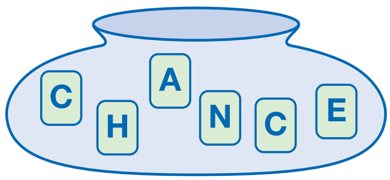

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HINT: The number of tiles in the bag is the number of possible outcomes.

Choose the best answer.

6. The probability of reaching into a drawer without looking and selecting a black sock is $\frac{5}{6}$. If there are a total of 24 socks in the drawer, how many socks in the drawer are black?

A. 4

B. 6

C. 15

D. 20

7. There are 10 comedies and 15 dramas in a bargain bin of DVDs at a store. If Marcus reaches in and selects a DVD without looking, what is the probability that he will select a drama?

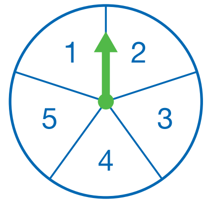
A. 0.15

B. 0.4

C. 0.6

D. 1.5

The spinner below on the right is divided into 5 equal-sized sections. Use the spinner for questions 8–10.



8. Write the sample space for this spinner.

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9. Identify P(even) and P(odd) for this spinner.

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10. If you spin the spinner 70 times, how many times would you expect the spinner to land on a section with a 2 on it? Show your work.

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Two number cubes, each with faces numbered 1 to 6, will be tossed at the same time. Use this situation for questions 11–13. Show or explain your work. Simplify, if possible.

11. What is the probability of tossing a sum of 7?

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12. What is the probability of tossing a sum greater than 7?

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13. What is the probability of tossing a sum less than 13?

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Solve. Simplify, if possible.

14. EXPLAIN Nadia tossed a number cube, with faces numbered 1 to 6, in the air 60 times. It came up 6 exactly 12 times. Determine the experimental probability and the theoretical probability of the cube coming up 6. Explain why it is reasonable that those two values are different.

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